Decoupling Limits of sGoldstino Modes in Global and Local Supersymmetry

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Abstract

We study the decoupling limit of a superheavy sgoldstino field in spontaneously broken $\mathcal{N}=1$ supergravity. After discussing sgoldstino decoupling in spontaneously broken globally supersymmetric theories in superspace, we analyze the same limit in supergravity. Our approach is based on Kähler superspace, which, among others, allows direct formulation of $\mathcal{N}=1$ supergravity in the Einstein frame and correct identifications of mass parameters. Allowing for a non-renormalizable Kähler potential in the hidden sector, the decoupling limit of a superheavy sgoldstino is identified with an infinite negative Kähler curvature. Constraints that lead to non-linear realizations of supersymmetry emerge as consequence of the equations of motion of the goldstino superfield when considering the decoupling limit. We also analyze supersymmetry breaking and sgoldstino decoupling in the case of many chiral multiplets. Finally, by employing superspace Bianchi identities, we identify the real chiral superfield, which will be the superconformal symmetry breaking chiral superfield that enters the conservation of the Ferrra-Zumino multiplet in the field theory limit of $\mathcal{N}=1$ supergravity.

1 Introduction

Supersymmetry is one of the most appealing cadidates for new physics. It has not been observed so far and thus, it should be broken at some high energy scale if it is realised at all. However, supersymmetry breaking is not an easy task. In the MSSM for example, supersymmetry breaking is employed by introducing soft breaking terms [1]. These terms are *ad hoc* masses for the superpartners of the SM particles, which nevertheless do not spoil the UV properties of the theory. In fact the MSSM includes all these soft breaking terms and one has to fit them into the observations.

From a more theoretical point of view, the origin of these soft terms should be explored. The common lore is that supersymmetry should be broken in a sector of the theory, not directly connected to the SM particles, the hidden sector. Then the breaking is mediated to the MSSM particles by some messenger fields-interactions [2,3]. In *Gravity Mediation* non-renormalizable Planck-suppressed interactions of the MSSM with the hidden sector are considered [4]. In *Gauge Mediation* the SM gauge interactions are responsible for giving rise to soft supersymmetry breaking terms [5]. Finally there exist more exotic scenarios like *Extra Dimensional Mediation* or *Anomaly Mediation*.

Whatever the nature of the mediation, the hidden sector should be studied on its own right. If it is a chiral multiplet that breaks supersymmetry, its highest component F will acquire a non-vanishing *vev*. There is a number of different scenarios for the origin of the supersymmetry breaking [1,2]. Let us note that higher derivative operators [6–9] may play an important role in hidden sector supersymmetry breaking. One of the most efficient methods for studying the phenomenology of the hidden sector is through the dynamics of the goldstino [10–23]. The latter is the fermionic component of the superfield that breaks supersymmetry. If the supersymmetry breaking scale is low, goldstino dynamics become increasingly important for low energy phenomenology [24–35]. In fact, if the SUSY breaking scale $\sqrt{f}$ is low with respect to Planck mass M_P ($\sqrt{f} \ll M_P$) as in gauge mediation, transverse gravitino couplings are of order M_P^{-1} and therefore are suppressed with respect to longitudinal gravitino couplings, which are of order $f^{-1/2}$. In this case, in the gravity decoupling limit, only the longitudinal gravitino component, i.e., the goldstino survives. Moreover, the highest component of the superfield to which the goldstino belongs, acquires a vev and breaks spontaneous the supersymmetry giving also mass to the sgoldstino (goldstino's superpartner). Therefore, at low energies, supersymmetry is spontaneous broken and after decoupling the sgoldstino (by making the latter superheavy) we are left with only the goldstino in the spectrum and a non-linear realised SUSY.

Recently new methods have been proposed in order to study goldstino couplings, and MSSM extensions that incorporate them have been constructed [32–39]. All this framework is based on the idea of constrained superfields [12, 13, 16] that introduce a non-linear supersymmetry representation for the

goldstino when its massive scalar superpartner is heavy and can be integrated out. Moreover, when one studies physics much lower than the MSSM soft masses scale, non-linear supersymmetry is realized on the SM particles as well, via the appropriate constraints. The constraint that enforces a non-linear supersymmetry realization for the goldstino reads

$$\Phi_{NL}^2 = 0. \tag{1.1}$$

As long as global supersymmetry is concerned, the origin and effects of this constraint have been extensively studied. Goldstino multiplet dynamics is also important in the case of the sgoldstino mass being comparable to the MSSM soft masses (for example see [40]). In this case there is no non-linear supersymmetry nevertheless.

In addition, it has been proven in [32] that in fact Φ_{NL} is proportional in the IR limit to the chiral superfield X that sources the violation of the conservation of the Ferrara-Zumino supercurrent $J_{\alpha\dot{\alpha}}$ [41]

$$\bar{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = D_\alpha X. \tag{1.2}$$

In the case of local supersymmetry, non-linear realizations are less studied in the supergravity context [13, 42, 43]. Trying to treat global and local supersymmetry uniformly, we will reformulate known results in the global superfield language, in order to pave the way for the extension to supergravity. We show that, by integrating out a supermassive sgoldstino, one finds a superfield constraint that leads to (1.1) as the only solution. This is done straight at the superfield level [44]. Turning to supergravity, the same method is followed [45], and by solving the superspace constraints, we find that the only consistent solution now is again given by (1.1). The lesson is that both cases can be treated uniformly, in the superspace language. This guaranties that the work done so far in non-linear supersymmetry in global superspace, is compatible with supergravity as well. In addition, we also identify the superfield in the $\mathcal{N} = 1$ supergravity, which turns out to be the chiral superfield X of (1.2) in the gravity decoupling limit. Here, the conservation of the Ferrara-Zumino multiplet $J_{\alpha\dot{\alpha}}$ in (1.2) is replaced by the consistency conditions of the Bianchi identities [45]

$$X_\alpha = \mathcal{D}_\alpha \mathcal{R} - \bar{\mathcal{D}}^{\dot{\alpha}} G_{\alpha\dot{\alpha}} \tag{1.3}$$

where $G_{\alpha\dot{\alpha}}$ and $\mathcal{R}$ are the usual supergravity superfields and $X_\alpha = -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8\mathcal{R})\mathcal{D}_\alpha K$ is the matter sector contribution.

The structure of the paper is as follows. In section 2 we discuss global supersymmetry. By identifying the decoupling limit of the sgoldstino as the limit of negative infinite Kähler curvature, we recover the non-linear supersymmetry constraints in the case of a single chiral superfield. The geneal case of many chiral superfields is also discussed. In section 3, the same method is followed for supergravity leading, to

the same results. We employ the *Kähler superspace* formalism in order to read the superfield equations of motion and the constraints in the Einstein frame. Finally, we conlude in section 4.

2 Supersymmetry in global superspace

We will describe here the IR limit of a spontaneous broken globally SUSY theory by assuming a superheavy sgoldstino. We will use superspace techniques and the results, although known [10]– [36], will allow us to set up notation and basic ideas. Moreover, the superspace formulation we will employ can be straightforwardly generalized to the local case and describes the sgoldstino decoupling in the $\mathcal{N}=1$ supergravity. To keep the discussion simple, we will consider first the case of a single chiral superfield. The case of more than one superfields will be worked out later in this section.

In superspace formalism, the most general single chiral superfield Lagrangian is given by

$$\mathcal{L}=\int d^4\theta\, K(\Phi,\bar{\Phi})\;+\;\left\{\int d^2\theta\, W(\Phi)+h.c.\right\} \tag{2.1}$$

where, $K(\Phi,\bar{\Phi})$ is the Kähler potential, a hermitian function of the chiral superfield, and $W(\Phi)$ is the superpotential, a holomorphic function of the chiral superfield. The superspace action (2.1) can be expressed as a pure D-term

$$\mathcal{L}=\int d^4\theta\left[K(\Phi,\bar{\Phi})-4\frac{DD}{16\Box}W(\Phi)-4\frac{\bar{D}\bar{D}}{16\Box}\bar{W}(\bar{\Phi})\right] \tag{2.2}$$

from which, the superspace equations of motion

$$-\frac{1}{4}\bar{D}\bar{D}K_\Phi+W_\Phi=0, \tag{2.3}$$

with $K_\Phi=\partial_\Phi K,\ W_\Phi=\partial_\Phi W$ easily follow. If supersymmetry is exact, the masses of all component fields of a given supermultiplet are degenerate. However, in the case of spontaneous breaking (or soft breaking) of supersymmetry, fermions and bosons are not any more degenerate and a mass splitting is introduced. To be precise, in the case of a single chiral multiplet, the scalar becomes massive in general while the fermion of the chiral multiplet remains strictly massless. This massless fermion changes by a shift under supersymmetry transformations i.e., it is the Goldstone fermion of the spontaneous broken supersymmetry, the goldstino. Its bosonic superpartner, the sgoldstino, is in general massive. Its mass M_{sg} in the case of a single chiral superfield can be deduced from the supertrace mass formula. For a general, non-renormalizable supersymmetric model where supersymmetry is spontaneously broken, the supertrace mass formula reads [43]

$$\mathrm{Str}M^2=\sum_J(-1)^{2J}(2J+1)M_J^2=-2R_{A\bar{A}}f\bar{f} \tag{2.4}$$

where $f = \langle F \rangle$ and

$$R_{A\bar{A}} = g^{A\bar{A}} R_{A\bar{A}A\bar{A}} \tag{2.5}$$

is the Ricci tensor of the scalar Kähler manifold evaluated at the vacuum expactation values of the scalars. Eq.(2.4) describes the mass splitting between the components of the supermultiplet. In the case of a single chiral superfield we are discussing, since the goldstino is always massless, the supertrace of the goldstino multiplet is just the square of the sgoldstino mass

$$M_{\rm sg}^2 = -R_{A\bar{A}} f \bar{f}. \tag{2.6}$$

We see that necessarily the scalar manifold should be a space of negative curvature in order to have non-tachyonic scalar excitations.

2.1 Sgoldstino decoupling in global supersymmetry

We are interested in those classes of models where the sgoldstino may become superheavy and decouples from the spectrum. In this case, it plays no role in the low energy effective theory, and its dynamics can be integrated out by its equations of motion. Essentialy, in order to be able to decouple consistently the sgoldstino degrees of freedom, one has to

1. consider the sgoldstino mass as the heavier scale in the problem, and
2. find consistent solutions for the equations of motion in that limit.

This is equivalent to taking the limit of infinitely heavy sgolstino and integrate its equations of motion, if possible, in this limit. This work has been done in component form earlier [16] and extended recently [36, 37]. We will implement the above procedure in superspace, where as we will see it is quite straightforward.

The limit of the infinitely heavy sgoldstino

$$2M_{sg}^2 = {\rm Str} M^2 \to \infty \tag{2.7}$$

is equivalent to the limit

$$R_{A\bar{A}A\bar{A}} \to -\infty. \tag{2.8}$$

Since

$$R_{A\bar{A}A\bar{A}} = \partial_{\bar{A}}\partial_A\partial_{\bar{A}}\partial_A K - \partial_{\bar{A}}\partial_A\partial_{\bar{A}} K \partial_A\partial_A\partial^A K\,, \tag{2.9}$$

in normal coordinates for the Kähler space in which $g_{A\bar{A}} = \delta_{A\bar{A}}$ and $\partial_i\partial_j\partial_k K = 0$ (for any $i, j = A, \bar{A}$), we have that the infinitely heavy sgoldstino is obtained in the limit

$$- \partial_{\bar{A}}\partial_A\partial_{\bar{A}}\partial_A K \to \infty. \tag{2.10}$$

By assuming that the vacuum expectation value of $A = \Phi\big|$ vanish[1], the general form of the Kähler potential

$$K(\Phi, \bar{\Phi}) = \sum_{mn} c_{mn}\Phi^m\bar{\Phi}^n \tag{2.11}$$

will have the following expansion in normal coordinates

$$K(\Phi, \bar{\Phi}) = \Phi\bar{\Phi} + c_{22}\bar{\Phi}^2\Phi^2 + \cdots \tag{2.12}$$

It is easy to see that in fact

$$c_{22} = \frac{1}{4}R_{A\bar{A}A\bar{A}} = \frac{1}{4}R_{A\bar{A}} \tag{2.13}$$

in normal coordinates. By using then (2.4,2.7), we get that the Kähler potential may be expressed in terms of the sgoldstino mass as

$$K(\Phi, \bar{\Phi}) = \Phi\bar{\Phi} - \frac{M_{sg}^2}{4|f|^2}\bar{\Phi}^2\Phi^2 + \cdots \tag{2.14}$$

where the dots stands for M_{sg}-independed terms and $f =< F >$ is the vev of the auxiliary field in the chiral multiplet. From the superspace equations of motion (2.3), one can easily isolate the contribution proportional to M_{sg}^2. Indeed,

$$K_\Phi = \bar{\Phi} - \frac{M_{sg}^2}{2|f|^2}\bar{\Phi}\Phi^2 + \cdots \tag{2.15}$$

so that (2.3) is written as

$$\frac{M_{sg}^2}{4|f|^2}\Phi\bar{D}\bar{D}\bar{\Phi}^2 \; + \; \Big(M_{sg}\text{−independed terms}\Big) = 0. \tag{2.16}$$

Therefore, in the $M_{sg} \to \infty$ limit, the field equations are turned just to the superspace constraint

$$\Phi\bar{D}\bar{D}\bar{\Phi}^2 = 0. \tag{2.17}$$

To explicitly solve (2.17), we note that it leads to three component equations

$$\Phi\bar{D}\bar{D}\bar{\Phi}^2| = 0, \quad D_\alpha(\Phi\bar{D}\bar{D}\bar{\Phi}^2)| = 0, \quad DD(\Phi\bar{D}\bar{D}\bar{\Phi}^2)| = 0. \tag{2.18}$$

[1] if not we may shift appropriately A so that $\langle A\rangle = 0$

By solving the first, we find

$$A = 0, \tag{2.19}$$

or

$$\bar{A} = \frac{\bar{\chi}\bar{\chi}}{2\bar{F}}. \tag{2.20}$$

Therefore the solution to (2.17) is [12, 32]

$$\Phi_{NL} = \frac{\chi\chi}{2F} + \sqrt{2}\theta\chi + \theta^2 F \tag{2.21}$$

which can be easily checked that it satisfies

$$\Phi_{NL}^2 = 0. \tag{2.22}$$

As a result, the sgoldstino can be safely decoupled in the $M_{sg} \to \infty$ limit as long as Φ satisfies (2.17), or equivalently (2.22).

Alternatively, (2.22) can also be derived by recalling that (2.17) for $\Phi \neq 0$ gives

$$\bar{D}\bar{D}\bar{\Phi}^2 = 0. \tag{2.23}$$

In superfield language, equation (2.23) means that Φ^2 is a complex linear multiplet. The latter are superfields L that satisfy $D^2 L = 0$. If in addition L is also chiral, then necessarily $L = 0$ (if $L| = 0$), i.e.,

$$D^2 L = 0 \quad \text{and} \quad DL = 0 \Rightarrow L = 0. \tag{2.24}$$

This is exactly our case: Φ^2 is both chiral and complex linear multiplet

$$\bar{D}\bar{D}\bar{\Phi}^2 = 0 \quad \text{and} \quad \bar{D}_{\dot{\alpha}}\bar{\Phi}^2 = 0. \tag{2.25}$$

Therefore, the only solution is $\Phi = \Phi_{NL}$ with Φ_{NL} satisfying $\Phi_{NL}^2 = 0$, since we have assumed that $< \Phi| >= 0$.

Let us recall now that if a theory possesses symmetries, there are conservation equations in addition to field equations. These conservations equations are consistent with the field equations and in fact can be derived from the latter. In the global supersymmetric case we are discussing, by multiplying (2.3) with $D_\alpha \Phi$ and making use of the identities

$$\begin{aligned} \bar{D}^{\dot{\alpha}}(2K_{\Phi\bar{\Phi}} D_\alpha \Phi \bar{D}_{\dot{\alpha}}\bar{\Phi}) &= \bar{D}^{\dot{\alpha}}(K_{\Phi\bar{\Phi}} D_\alpha \Phi \bar{D}_{\dot{\alpha}}\bar{\Phi} - K_\Phi \{D_\alpha, \bar{D}_{\dot{\alpha}}\}\Phi) + D_\alpha \Phi \bar{D}\bar{D} K_\Phi, && (2.26) \\ \frac{1}{3} D_\alpha \bar{D}\bar{D} K &= \frac{1}{2}\bar{D}^{\dot{\alpha}}\{D_\alpha, \bar{D}_{\dot{\alpha}}\}K + \frac{1}{6}\bar{D}^{\dot{\alpha}}[D_\alpha, \bar{D}_{\dot{\alpha}}]K && (2.27) \end{aligned}$$

we arrive in the supercurrent conservation law

$$\bar{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = D_\alpha X \tag{2.28}$$

with

$$X = 4W - \frac{1}{3}\bar{D}^2 K, \tag{2.29}$$

$$J_{\alpha\dot{\alpha}} = 2K_{\Phi\bar{\Phi}} D_\alpha \Phi \bar{D}_{\dot{\alpha}} \bar{\Phi} - \frac{2}{3}[D_\alpha, \bar{D}_{\dot{\alpha}}]K. \tag{2.30}$$

We easily recognize that $J_{\alpha\dot{\alpha}}$ is the Ferrara-Zumino multiplet [41], which is a Lorentz vector, real superfield and contains the supersymmetric current, the energy-momentum tensor as well as the R-current in its superspace components, and X is an appropriate chiral superfield. The superfield X is therefore written as

$$X = 4W - \frac{1}{3}\Phi \bar{D}^2 \Phi + \frac{M_{sg}^2}{12|f|^2}\Phi^2 \bar{D}^2 \bar{\Phi}^2 + \dots \tag{2.31}$$

Therefore, in the $M_{sg} \to \infty$ limit, X is divergent except if $\Phi^2 \bar{D}^2 \bar{\Phi}^2 = 0$ is satisfied. This is equivalent to the constraint of eq.(2.17). Using the field equation for the constraint superfield Φ, we get that

$$X = \frac{8}{3} f \Phi. \tag{2.32}$$

We recognize here the relation found by Seiberg and Komargodski [32] for the IR relation of the chiral superfield Φ to the chiral superfield X, which enters in the conservation equation of the Ferrara-Zumino multiplet.

In addition, the Kähler potential for Φ_{NL} will effectively always be reduced to

$$\tilde{K} = \bar{\Phi}_{NL} \Phi_{NL}, \tag{2.33}$$

and the superpotential will be

$$\tilde{W} = f\, \Phi_{NL}. \tag{2.34}$$

Then one recovers the superspace Lagrangian

$$\tilde{\mathcal{L}} = \int d^4\theta\, \bar{\Phi}_{NL} \Phi_{NL} + \int d^2\theta\, f\, \Phi_{NL} + h.c. \tag{2.35}$$

where Φ_{NL} satsifies

$$\Phi_{NL}^2 = 0. \tag{2.36}$$

After superintegration, solving the auxiliary equations of motion and plugging them back into the action, one finds [12, 32]

$$\tilde{\mathcal{L}} = -f^2 + i\partial_\mu \bar{\chi}\bar{\sigma}\chi + \frac{1}{4f^2}\bar{\chi}^2\partial^2\chi^2 - \frac{1}{16f^6}\chi^2\bar{\chi}^2\partial^2\chi^2\partial^2\bar{\chi}^2. \tag{2.37}$$

Note that (2.37) is equivalent to the well known Akulov-Volkov Lagrangian [10], which describes goldstino dynamics and non-linear supersymmetry.

2.2 The decoupling for general chiral models

Let us consider a chiral model with $n+1$ superfields, that also exhibits spontaneous supersymmetry breaking

$$\mathcal{L} = \int d^4\theta\, K(\Phi^i, \bar{\Phi}^{\bar{j}}) + \left\{ \int d^2\theta W(\Phi^i) + h.c. \right\}, \qquad i = 1, \ldots, n+1. \tag{2.38}$$

The superfield equations read

$$-\frac{1}{4}\bar{D}\bar{D}K_i + W_i = 0 \tag{2.39}$$

where

$$K_i = \frac{\partial K(\Phi, \bar{\Phi})}{\partial \Phi^i}, \quad W_i = \frac{\partial W(\Phi)}{\partial \Phi^i}. \tag{2.40}$$

The superfield equations (2.38) correspond to three component equations each: one for the physical scalars, one for the physical fermions and one for the auxiliary scalars. In particular, the scalar field equations can be written as

$$W_{ij}f^j = 0, \tag{2.41}$$

$$g_{i\bar{j}}\bar{f}^{\bar{j}} + W_i = 0 \tag{2.42}$$

with $f^i =< F^i >$ and all quantities in (2.41) and (2.42) are refering to their vacuum values.

Since supersymmetry is broken, the fermionic shifts will not vanish in the vacuum

$$< \delta\chi_i >= -f_i\epsilon. \tag{2.43}$$

By an appropriate rotation of χ_i, we can define new fermionic fields $\tilde{\chi}_i$

$$\tilde{\chi}_i = R_{ij}\chi_j \tag{2.44}$$

where R_{ij} is an appropriate matrix such that the non-zero fermionic shifts are along a specific direction, which we will call it ("0")

$$< \delta\tilde{\chi}_0 >= -f\epsilon\,, \qquad < \delta\tilde{\chi}_a >= 0\,, \quad a = 1, \ldots, n. \tag{2.45}$$

Clearly $\delta\tilde{\chi}_0$ is the goldstino, defined as

$$\delta\tilde{\chi}_0 = R_{0i}\delta\chi_i \tag{2.46}$$

whereas the rest of the modes are given by

$$\delta\tilde{\chi}_a = R^{ai}\delta\chi_i. \tag{2.47}$$

The matrix R_{ij} is orthogonal and has been chosen to satisfy

$$R_{ai}f_i = 0\,. \tag{2.48}$$

It can be proven that if (2.48) is satisfied, then $R_{0i} = f_i/|f|\,,\quad (|f|^2 = f_i\bar{f}_i)$ so that the goldstino is

$$\delta\tilde{\chi}_0 = \frac{f_i}{|f|}\delta\chi_i\,. \tag{2.49}$$

Note that instead of rotating χ_i's, we could have rotated the original superfields Φ^i so that the goldstino belongs to the $\tilde{\Phi}^0$ superfield, which is a linear combination of the original fields. According to (2.49), $\tilde{\Phi}^0$ is

$$\tilde{\Phi}_0 = \frac{f_i}{|f|}\Phi^i. \tag{2.50}$$

The rest of the superfields will be given according to

$$\tilde{\Phi}_a = R_{ai}\Phi^i. \tag{2.51}$$

For example in the case of two superfields Φ_1, Φ_2 we find that the goldstino belongs to

$$\tilde{\Phi}_0 = \frac{1}{|f|}\left(f_1\Phi_1 + f_2\Phi_2\right) \tag{2.52}$$

whereas the second superfield is

$$\tilde{\Phi}_1 = \frac{1}{|f|}\left(-|f_2|e^{i\varphi_1}\Phi_1 + |f_1|e^{i\varphi_2}\Phi_2\right)\,, \quad \varphi_i = \arg(f_i). \tag{2.53}$$

For three superfields Φ_1, Φ_2, Φ_3, we find that again the goldstino belongs to

$$\tilde{\Phi}_0 = \frac{1}{|f|}\left(f_1\Phi_1 + f_2\Phi_2 + f_3\Phi_3\right) \tag{2.54}$$

and the rest of the superfields are

$$\begin{aligned}
\tilde{\Phi}_1 &= \frac{1}{\sqrt{|f_1|^2 + |f_2|^2}}\left(|f_2|e^{i\varphi_3}\Phi_3 - |f_3|e^{i\varphi_2}\Phi_2\right) \\
\tilde{\Phi}_2 &= \frac{1}{\sqrt{|f_1|^2 + |f_2|^2}}\left\{\frac{|f_1|}{|f|}\left(\bar{f}_2\Phi_2 + \bar{f}_3\Phi_3\right) - \frac{|f_2|^2 + |f_3|^2}{|f|}e^{-i\phi_1}\Phi_1\right\}.
\end{aligned} \tag{2.55}$$

It should be noted that $\tilde{\Phi}_a$ are not uniquely determined since there is an $O(n)$ freedom of rotating the directions perpendicular to the goldstino direction. Thus, there are $n(n-1)/2$ parameters that can be fixed at will, leaving only n independed parameters to be determined by the equation (2.51) for the $n+1$ directions $\delta\chi_i$. Therefore, after doing this superfield redefinitions, the Kähler potential and the superpotential will be functions of the new fields $\tilde{\Phi}_0, \tilde{\Phi}_a$, i.e., $K(\tilde{\Phi}_0, \tilde{\Phi}_a, \tilde{\Phi}_0^\dagger, \tilde{\Phi}_a^\dagger)$ and $W(\tilde{\Phi}_0, \tilde{\Phi}_a)$.

In the case of many chiral multiplets, we cannot read off the sgoldstino mass just by inspection of the supertrace formula, as there are now more scalars contibuting to the latter. Instead, we have to look into the mass matrix itself. Following the standard procedure [43], we find that the scalar mass matrix is given in normal coordinates for the scalar manifold by

$$M_B^2 = \begin{pmatrix} -K_{ik\bar{j}\bar{l}}\bar{f}^{\bar{l}} f^k + W_{ik}\bar{W}_{\bar{r}\bar{j}}K^{k\bar{r}} & -W_{ijk}f^k \\ -W_{\bar{r}\bar{k}\bar{l}}\bar{f}^{\bar{l}} & -K_{j\bar{l}\bar{i}\bar{k}}\bar{f}^{\bar{k}} f^l + W_{\bar{i}\bar{k}}\bar{W}_{rj}K^{r\bar{k}} \end{pmatrix} \tag{2.56}$$

where now all quantities are refered to the new redefined fields. In particular, the vacuum field equations (2.41) turned then out to be

$$W_{0a}f = 0, \tag{2.57}$$

which leads to

$$W_{0a} = 0, \ \text{for every } a. \tag{2.58}$$

Moreover in order to forbid mass mixing of the form $\tilde{A}^a\tilde{A}^b$, we assume that

$$W_{ab0} = 0. \tag{2.59}$$

Eventually, the important part of the boson mass matrix becomes

$$\begin{aligned} (M_B^2)_{0\bar{0}} &= -K_{00\bar{0}\bar{0}}\bar{f}f \\ (M_B^2)_{0\bar{b}} &= -K_{00\bar{b}\bar{0}}\bar{f}f \\ (M_B^2)_{a\bar{0}} &= -K_{a0\bar{0}\bar{0}}\bar{f}f \\ (M_B^2)_{a\bar{b}} &= -K_{a0\bar{b}\bar{0}}\bar{f}f + W_{ac}\bar{W}_{\bar{d}\bar{b}}K^{c\bar{d}}. \end{aligned} \tag{2.60}$$

Since the mass matrix is not diagonal it can be seen that the sgoldstino mass will have effect on the masses of the rest of the scalar particles as well. For two chiral multiplets, for example, since the masses are just the eigenvalues of the scalar mass matrix we have

$$\begin{aligned} m_i^2 &= -\frac{1}{2}(K_{00\bar{0}\bar{0}}\bar{f}f + K_{01\bar{0}\bar{1}}\bar{f}f - W_{1c}\bar{W}_{\bar{d}\bar{1}}K^{c\bar{d}}) \\ &\pm \frac{1}{2}\sqrt{(-K_{00\bar{0}\bar{0}}\bar{f}f + K_{10\bar{1}\bar{0}}\bar{f}f - W_{1c}\bar{W}_{\bar{d}\bar{1}}K^{c\bar{d}})^2 + 4|K_{10\bar{0}\bar{0}}\bar{f}f|^2}. \end{aligned} \tag{2.61}$$

Depending on the relative magnitude of the various contributions inside the masses (2.61), one finds different implications [38]. Let us only study here the limit

$$K_{00\bar{0}\bar{0}} \to -\infty \tag{2.62}$$

then

$$\begin{aligned} m_1^2 &= -K_{00\bar{0}\bar{0}}\bar{f}f \\ m_2^2 &= -K_{01\bar{0}\bar{1}}\bar{f}f + W_{1c}\bar{W}_{\bar{d}\bar{1}}K^{c\bar{d}} \end{aligned} \tag{2.63}$$

thus it is clear that this will only correspond to a superheavy sgoldstino. It seems like the limit (2.62) makes the Kähler potential acquire a separable form, and there is no mixing any more. This happens because the terms that contribute to the mixing of the masses will be suppressed with $K_{00\bar{0}\bar{0}}$.

3 Goldstinos in Supergravity

In the standard $\mathcal{N} = 1$ superspace forlmulation of supergravity, one is forced to perform a Weyl rescaling to the action in order to write the theory in the Einstein frame. Here, we should write the superspace action directly in the Einstein frame since we want to correctly identify the masses to be send to infinity. This can be quite complicated especially in looking for consistent solutions to the field equations in this limit. However, a more appropriate formalism exists which allows to an easy identification of the parameters we should take large as well as to a straigthforward way to discuss equations of motion in the limit under consideration. This is the Kähler superspace formalism which we will briefly present below. For a detailed description, one may consult for example [45–47].

3.1 Supergravity in Kähler superspace

In the conventional superspace approach to supergravity, the Lagrangian describing gravity coupled to matter would be (ignoring superpotential for the moment)

$$\mathcal{L}_{\mathrm{F}} = \int d^2\Theta 2\mathcal{E}\left\{\frac{3}{8}(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R})e^{-\frac{1}{3}K(\Phi,\bar{\Phi})}\right\} + h.c. \tag{3.1}$$

where $2\mathcal{E}$ is the superspace chiral density and the new Θ variables span only the chiral superspace. An equivalent way to write the action (3.1) is

$$\mathcal{L}_{\mathrm{D}} = -3\int d^4\theta E e^{-\frac{1}{3}K(\Phi,\bar{\Phi})}, \tag{3.2}$$

where now E is the full superspace density and θ are to be integrated over the full superspace. Both actions (3.1,3.2) can equavalently be used in order to build invariant theories in superspace. Note that $\mathcal{E}$ and E,

both have the vierbein determinant in their lowest component. As usual $\mathcal{R}$ represents the supergravity chiral superfield which contains the Ricci scalar in its highest component. Direct calculation of (3.2) in component form shows that the theory is actually expressed in an unconventional Jordan frame. Of course a Weyl rescaling may be performed in order to bring the theory in the standard Einstein frame. Nevertheless, it is possible to perform this rescaling at the superspace level by considering

$$\begin{aligned} E'^{\ a}_{M} &= e^{-\frac{1}{6}K(\Phi,\bar{\Phi})}E^{a}_{M} \\ E'^{\ \alpha}_{M} &= e^{-\frac{1}{12}K(\Phi,\bar{\Phi})}\left[E^{\alpha}_{M} - \frac{i}{12}E^{b}_{M}(\epsilon\sigma_b)^{\alpha}{}_{\dot{\alpha}}\bar{\mathcal{D}}^{\dot{\alpha}}K(\Phi,\bar{\Phi})\right] \\ E'_{M\dot{\alpha}} &= e^{-\frac{1}{12}K(\Phi,\bar{\Phi})}\left[E_{M\dot{\alpha}} - \frac{i}{12}E^{b}_{M}(\epsilon\bar{\sigma}_b)_{\dot{\alpha}}{}^{\alpha}\mathcal{D}_{\alpha}K(\Phi,\bar{\Phi})\right] \end{aligned} \tag{3.3}$$

where E^M_A is the superspace frame, containing the gravitino and the vierbein in the appropriate lowest components. This redefinition will change the structure of the whole superspace including the Bianchi identity solutions and the superspace derivatives. Most importantly, the superspace geometry will receive contributions at the same time from the matter and supergravity fields in a unified way. The Lagrangian (3.2) now becomes in the new superspace frame (erasing the primes for convenience)

$$\mathcal{L}_{\text{Dnew}} = -3\int d^4\theta E \tag{3.4}$$

This form now contains the properly normalized supergravity action coupled to matter. The interested reader should consult an extensive review on the subject [45]. Since we also wish to include a superpotential, the appropriate contribution will be given by adding to (3.4) the appropriatelly rescaled superpotential W so that the full Lagrangian will be given by

$$\mathcal{L}_{superpotential} = -3\int d^4\theta E + \left\{\int d^4\theta \frac{E}{2\mathcal{R}}e^{K/2}W + h.c.\right\}. \tag{3.5}$$

In this new framework, Kähler transformations, generated by holomorphic functions $\mathcal{F}$, are expressed as field dependent transformations gauged by a composite $U_K(1)$ vector B_A. The respective charge now is referred to as "chiral weight" and a superfield $\varPhi$ of chiral weight $w(\varPhi)$ transforms as

$$\varPhi \to \varPhi e^{-\frac{i}{2}w(\varPhi)\mathrm{Im}\mathcal{F}} \tag{3.6}$$

while gauge covariant superspace derivatives are defined as

$$\mathcal{D}_A\varPhi = E_A^{\ M}\partial_M\varPhi + w(\varPhi)B_A\varPhi \tag{3.7}$$

and all component fields are understood to be defined appropriately via projection as usual but now with the use of these Kähler-superspace derivatives. The composite connection superfields are

$$\begin{aligned} B_\alpha &= \frac{1}{4}\mathcal{D}_\alpha K \\ \bar{B}^{\dot\alpha} &= -\frac{1}{4}\bar{\mathcal{D}}^{\dot\alpha}K \\ B_a &= \frac{1}{4}(\partial_i K)\mathcal{D}_a\Phi^i - \frac{1}{4}(\partial_{\bar j}K)\mathcal{D}_a\bar\Phi^{\bar j} + \frac{3i}{2}\mathcal{G}_a + \frac{i}{8}g_{i\bar j}\bar\sigma^{\dot\alpha\alpha}(\mathcal{D}_\alpha\Phi^i)\bar{\mathcal{D}}_{\dot\alpha}\bar\Phi^{\bar j} \end{aligned} \tag{3.8}$$

so that

$$B_a \to B_a - \mathcal{D}_a(-\frac{i}{2}\mathrm{Im}\mathcal{F}). \tag{3.9}$$

It turns out that the invariant Lagrangian containing both (3.4) and (3.5) depends only on the generalized Kähler potential

$$e^G = e^{K(\Phi,\bar\Phi)}W(\Phi)\bar{W}(\bar\Phi). \tag{3.10}$$

By taking into account the chiral weights of the gravity sector and performing a Kähler transformation with parameter $\mathcal{F} = \ln W$, we find that the final expression for the most general coupling of matter to supergravity is

$$\mathcal{L} = \int d^4\theta E\left[-3 + \frac{1}{2\mathcal{R}}e^{\frac{G}{2}} + \frac{1}{2\bar{\mathcal{R}}}e^{\frac{G}{2}}\right]. \tag{3.11}$$

It sould be stressed that this form of the action is completely equivalent to the standard $\mathcal{N} = 1$ superspace formulation (3.1) to which is related by appropriate redefinitions of the superspace frames.

3.2 Sgoldstino decoupling

As in the case of global supersymmetry, we are interested in the equations of motion and the mass supertrace. The superfield equations of motion as follow from the action (3.11) are [46]

$$\mathcal{R} = \frac{1}{2}e^{\frac{G}{2}}, \tag{3.12}$$

$$\mathcal{G}_a + \frac{1}{8}G_{\Phi\bar\Phi}\bar\sigma_a^{\dot\alpha\alpha}\mathcal{D}_\alpha\Phi\bar{\mathcal{D}}_{\dot\alpha}\bar\Phi = 0, \tag{3.13}$$

$$(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R})G_\Phi = 0. \tag{3.14}$$

On the other hand, for a general supergravity model with only one chiral multiplet the supertrace is given by [44]

$$\mathrm{Str}M^2 = -2R_{A\bar A}f\bar f, \tag{3.15}$$

which means that in the limit of infinite negative Kähler curvature the sgoldstino will become superheavy and can consistently be integrated out. Indeed, (3.15) is explicitly written as

$$M_{sg}^2 = 2m_{3/2}^2 - R_{A\bar{A}} f\bar{f}. \tag{3.16}$$

Therefore, for finite gravitino mass $m_{3/2}$, the infinite curvature limit

$$R_{A\bar{A}A\bar{A}} \to -\infty \tag{3.17}$$

is equivalent to superheavy sgoldstinos. Again, in normal coordinates

$$R_{A\bar{A}A\bar{A}} = \partial_{\bar{A}}\partial_A\partial_{\bar{A}}\partial_A K = \partial_{\bar{A}}\partial_A\partial_{\bar{A}}\partial_A G \tag{3.18}$$

and therefore with

$$G \supset \frac{2m_{3/2}^2 - M_{sg}^2}{4|f|^2}\Phi^2\bar{\Phi}^2 + \cdots \tag{3.19}$$

the decoupling limit we are after is again $M_{sg}^2 \to \infty$. Taking into account that the Kähler curvature $M_{sg}^2/4|f|^2$ will dominate the equations of motion and following the same reasoning as in the global supersymmetric case, we get from (3.14)

$$\Phi(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R})\bar{\Phi}^2 = 0. \tag{3.20}$$

This constraint is the curved superspace analogue of (2.17). In order to solve it, we take into account that $\Phi(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R})\bar{\Phi}^2$ is a chiral superfield, and we will once again start from its lowest component, namely

$$\Phi(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R})\bar{\Phi}^2| = 0. \tag{3.21}$$

This is written, for

$$\Phi = A + \sqrt{2}\Theta\chi + \Theta\Theta F \tag{3.22}$$

and

$$\mathcal{R}\Big| = -\frac{1}{6}M \tag{3.23}$$

as

$$AM\bar{A}^2 - 24A\bar{A}\bar{F} + 12A\bar{\chi}\bar{\chi} = 0. \tag{3.24}$$

This equation has three solutions

$$\begin{aligned}
\bar{A}_0 &= 0, && (3.25)\\
\bar{A}_1 &= \frac{\bar{\chi}\bar{\chi}}{2\bar{F}}, && (3.26)\\
\bar{A}_2 &= \frac{24\bar{F}}{M} - \frac{\bar{\chi}\bar{\chi}}{2\bar{F}}. && (3.27)
\end{aligned}$$

The first solution $A = 0$ is the trivial and we will not consider it. The second solution (3.26) is the $\Phi^2 = 0$ we already encounter in the global susy case. The third solution (3.27) corresponds to $\Phi^2 \neq 0$ and can only be realized as long as the auxiliary field of supergravity M is non vanishing ($M \neq 0$). However, from the equation (3.12) we get

$$\mathcal{R} = \frac{1}{2} e^{\frac{G}{2}} = \frac{1}{2} e^{-\frac{M_{sg}^2}{8|f|^2}\Phi^2\bar{\Phi}^2 + \cdots}, \tag{3.28}$$

where only the dominant term was explicitly written in the exponent in the right hand side. Now, in the $M_{sg}^2 \to \infty$ limit, the right hand side goes to zero exponentialy fast so that for $\Phi^2 \neq 0$

$$\mathcal{R} = 0 \quad \text{for} \quad M_{sg}^2 \to \infty. \tag{3.29}$$

Therefore also $M = -6\mathcal{R}| = 0$ and the third solution (3.27) cannot be realized. As a result, the only solution to the constraint (3.20) is the second one (3.26), or in other words the familiar

$$\Phi^2 = 0. \tag{3.30}$$

This constraint leads to

$$e^{\frac{M_{sg}^2}{8|f|^2}\Phi^2\bar{\Phi}^2}|_{\Phi^2=0} = 1 \tag{3.31}$$

and thus, the divergent part of (3.12) completely decouples! Moreover, $\Phi^2 = 0$ also satisfies

$$\mathcal{D}_\alpha \Phi \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\Phi}^2 = 0 \tag{3.32}$$

which is the field equation (3.13) in the $M_{sg}^2 \to \infty$ limit. As a result, we have again arrived to the constraint (3.30) as the only viable and consistent condition for the decoupling of the sgoldstino. Something very interesting has happened here. First, we notice that the solution $\Phi^2 \neq 0$, even if it might exist from (3.20), would lead to inconsistencies when combined with (3.12). Second, by picking the solution (3.26), the potential divergent terms of (3.28) completely decouple, leaving only the condition $\bar{A} = \frac{\bar{X}\bar{X}}{2F}$ without imposing any constraint on the supergravity auxiliary M. Again, we should make clear that the constraint $\Phi^2 = 0$ has emerged as the only valid solution of the more fundamental constraints (3.20) and (3.32), when combined with (3.12).

3.3 Supercurrent and sgoldstino decoupling

In order to discuss the relation of supersymmetry breaking to conservation laws, let us explore the decoupling limit of the supergravity sector. The supergravity equations of motion (3.12) and (3.13) in

superspace, after restoring dimensions with compensating powers of M_P and returning to the Kähler frame where everything is expressed in terms of K and W, are written as

$$\mathcal{R} = \frac{1}{M_P^2}\frac{1}{2}We^{\frac{K}{2M_P^2}}, \tag{3.33}$$

$$\mathcal{G}_a + \frac{1}{M_P^2}\frac{1}{8}g_{i\bar{j}}\bar{\sigma}_a^{\dot{\alpha}\alpha}\mathcal{D}_\alpha\Phi^i\bar{\mathcal{D}}_{\dot{\alpha}}\bar{\Phi}^{\bar{j}} = 0. \tag{3.34}$$

Gravity decouples in the limit

$$M_P \to \infty \tag{3.35}$$

and from (3.33) and (3.34) we have

$$\mathcal{R} \to 0, \tag{3.36}$$

$$\mathcal{G}_a \to 0. \tag{3.37}$$

We note that this is the limit even when $W/M_P =$ finite, which is another possible limit [42] for gauge mediated SUSY breaking scenarios. The fact that these supergravity superfields should vanish can be also understood from the algebra of supergravity when compared to supersymmetry. For example, the global commutation relation (for $w(\Phi^i) = 0$)

$$[\bar{D}_{\dot{\alpha}}, D_a]\Phi^i = 0, \tag{3.38}$$

in supergravity becomes

$$[\bar{\mathcal{D}}_{\dot{\alpha}}, \mathcal{D}_a]\Phi^i = -i\mathcal{R}\sigma_{\alpha\dot{\alpha}}\mathcal{D}^\alpha\Phi^i \tag{3.39}$$

thus in order to recover the global supersymmetry algebra the superfield $\mathcal{R}$ should vanish.

Let us now derive the analog of equation (2.28) in curved superspace. By using the consistency conditions of the Bianchi identities [45]

$$X_\alpha = M_P^2\mathcal{D}_\alpha\mathcal{R} - M_P^2\bar{\mathcal{D}}^{\dot{\alpha}}G_{\alpha\dot{\alpha}} \tag{3.40}$$

with

$$X_\alpha = -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8\mathcal{R})\mathcal{D}_\alpha K \tag{3.41}$$

and the equations of motion, we find

$$\bar{\mathcal{D}}^{\dot{\alpha}}\mathcal{J}_{\alpha\dot{\alpha}} = \mathcal{D}_\alpha\mathcal{X} - \frac{16}{3}\mathcal{R}\mathcal{D}_\alpha K + \frac{2}{3}\mathcal{G}_{\alpha\dot{\alpha}}\bar{\mathcal{D}}^{\dot{\alpha}}K \tag{3.42}$$

with

$$\mathcal{J}_{\alpha\dot{\alpha}} = 2g_{i\bar{j}}\mathcal{D}_\alpha\Phi^i\bar{\mathcal{D}}_{\dot{\alpha}}\bar{\Phi}^{\bar{j}} - \frac{2}{3}[\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}]K \tag{3.43}$$

and

$$\mathcal{X} = 4We^{\frac{K}{2M_P^2}} - \frac{1}{3}\bar{\mathcal{D}}\bar{\mathcal{D}}K. \tag{3.44}$$

The extra terms compared to (2.28) arise due to commutation relations like (3.39), and should vanish when supergravity is decoupled.

Now we take the decoupling limit of supergravity ($M_P \to \infty$) with ($\mathcal{R} \to 0,\ \mathcal{G}_a \to 0$) and find exactly the same formula as the global case. As a final comment let us note that now, after the decoupling of supergravity, the superfield X is

$$\mathcal{X} \to X = 4W - \frac{1}{3}\bar{D}\bar{D}K. \tag{3.45}$$

3.4 Non-linear supersymmetry realization inside supergravity

Once the constraint $\Phi_{NL}^2 = 0$ is imposed, the generalized Kähler potential will be reduced to

$$\tilde{G} = \bar{\Phi}_{NL}\Phi_{NL} + a\Phi_{NL} + a\bar{\Phi}_{NL} + b. \tag{3.46}$$

In this case, the supergravity theory in component form is presented in the Appendix. Up to two fermion terms, which is important for the rest of our discussion, the Lagrangian is explicitly written as

$$\begin{aligned} e^{-1}\mathcal{L} &= -\frac{1}{2}R + \frac{1}{2}\varepsilon^{mnpq}\bar{\psi}_m\bar{\sigma}_n\partial_p\psi_q - \frac{1}{2}\varepsilon^{mnpq}\psi_m\sigma_n\partial_p\bar{\psi}_q - \frac{i}{2}\chi\sigma^m\partial_m\bar{\chi} - \frac{i}{2}\bar{\chi}\bar{\sigma}^m\partial_m\chi \\ &+ \frac{i}{\sqrt{2}}e^{b/2}a(\bar{\psi}_m\bar{\sigma}^m\chi) + \frac{i}{\sqrt{2}}e^{b/2}a(\psi_m\sigma^m\bar{\chi}) - e^{b/2}\bar{\psi}_m\bar{\sigma}^{mn}\bar{\psi}_n - e^{b/2}\psi_m\sigma^{mn}\psi_n \\ &+ (3-a^2)e^b - e^{b/2}\bar{\chi}^2 - e^{b/2}\chi^2. \end{aligned} \tag{3.47}$$

The constant term in (4.2) tells us it should be possible to find a supersymmetry transformation with a field dependent parameter $\xi(\chi, \psi_m, e_m^a)$, such that $\delta_\xi\chi = -\chi$, or

$$i\sqrt{2}(\bar{\xi}\bar{\sigma}^m\epsilon)_\alpha(\partial_m A - \frac{1}{\sqrt{2}}\psi_m\chi) + (\sqrt{2}F + \frac{1}{4\sqrt{2}}\chi^2[\frac{\bar{\chi}^2}{2\bar{F}} + a])\xi_\alpha - \frac{a\ \bar{\xi}\bar{\chi}}{2\sqrt{2}}\chi_\alpha = -\chi_\alpha \tag{3.48}$$

and thus the goldstino can be gauged away. Equation (3.48) is solved iteratively and is guaranteed to have a solution. Eventually, the final Lagrangian (in the unitary gauge) becomes

$$\begin{aligned} e^{-1}\mathcal{L} &= -\frac{M_P^2}{2}R + \frac{1}{2}\varepsilon^{mnpq}\bar{\psi}_m\bar{\sigma}_n[\partial_p\psi_q + \psi_q\omega_p] - \frac{1}{2}\varepsilon^{mnpq}\psi_m\sigma_n[\partial_p\bar{\psi}_q + \bar{\psi}_q\omega_p] \\ &+ M_P^4(3-a^2)e^b - M_Pe^{\frac{b}{2}}\psi_m\sigma^{mn}\psi_n - M_Pe^{\frac{b}{2}}\bar{\psi}_m\bar{\sigma}^{mn}\bar{\psi}_n \end{aligned} \tag{3.49}$$

where we have also restored the dimensions of the fields involved setting

$$\tilde{G} = \frac{1}{M_P^2}\Phi_{NL}\bar{\Phi}_{NL} + \frac{a}{M_P}\Phi_{NL} + \frac{\bar{a}}{M_P}\bar{\Phi}_{NL} + b. \tag{3.50}$$

Interpreting the last two terms in (3.49) as gravitino mass term we need flat Minkowski background. This is possible if the the relation

$$a^2 = 3 \tag{3.51}$$

is satisfied. In this case we get that the gravitino mass is

$$m_{3/2} = M_P e^{\frac{b}{2}} \tag{3.52}$$

which fixes the parameter b in the superpotential

$$b = 2\ln(\frac{m_{3/2}}{M_P})\,. \tag{3.53}$$

In addition, the susy breaking scale, the Planck mass and the gravitino mass are related through

$$< F >= -\sqrt{3}M_P m_{3/2}. \tag{3.54}$$

Even though we have implied previously that the cosmological constant has to be zero, and used it as a fact, we never explicitly imposed it until now. As usual, the goldstino has not disappeared, it is eaten by the gravitino, and it is expected to play some role in high energy scattering amplitudes in the low energy effective theories [49–52].

Finally, let us also briefly discuss the case of many chiral multiplets coupled to supergravity, which exhibits spontaneously SUSY breaking and a vanishing cosmological constant. In terms of the generalised Kähler potential, $G = K + \ln P + \ln\bar{P}$, and in normal coordinates, the scalar mass matrix is

$$M_B^2 = \begin{pmatrix} G_{ik}G^{k\bar{l}}G_{\bar{l}\bar{j}} - G_{i\bar{j}k\bar{l}}G^kG^{\bar{l}} + G_{i\bar{j}} & 2G_{ij} \\ 2G_{\bar{i}\bar{j}} & G_{\bar{i}\bar{k}}G^{\bar{k}l}G_{lj} - G_{\bar{i}j\bar{k}l}G^{\bar{k}}G^l + G_{\bar{i}j} \end{pmatrix} m_{3/2}^2 \tag{3.55}$$

where everything is evaluated on the ground state. The discussion of section 2.2 still applies here as well and we may again after appropriate rottaion of the chiral superfields, we may arrange such that the goldstino to belong to a specific multiplet $\tilde{\Phi}_0$. Again, the masses of the scalars of the rest of the multiplets mix with the sgoldstino mass; the sgoldstino decoupling limit (3.17) nevertheless will force the Kähler potential to have a separable form, and no mixing will be present.

4 Conclusions

In this work we explored the decoupling limit of sgoldstinos in a spontaneously broken SUSY theories. This decoupling was implemented by considering large mass values for the sgoldstino (in fact the infinite mass limit). We used superspace techniques as they allowed for a unified treatment of the spontaneous breaking of SUSY both in local and global supersymmetric cases. The motivation of this study was twofold: first to check if the constraint superfield formalism employed in the global supersummetry still works in supergravity as well and second, to correctly identify in supergavity the chiral superfield that enters in the conservation of the Ferrara-Zumino multiplet and which accomodates the goldstino in global supersymmetry.

The way to approach these targets was to reformulate the goldstino dynamics in global supersymmetry but now in a language appropriate for supergravity. First we have identified the sgoldstino mass in both cases, and found the decoupling limit (supermassive sgoldstino) to be the limit of infinite negative Kähler curvature. Then we impose this limit to the superfield equations of motion and in the case of supersymmetry we found the constraint ($\Phi\bar{D}^2\bar{\Phi}^2 = 0$) which is solved by $\Phi^2 = 0$ as expected. In the case of supergravity, the super-covariant form of the more general constraint emerges, but again with the same single consistent solution. Thus, the superspace constraint $\Phi^2 = 0$ for the goldstino, when the sgoldstino is supermassive, holds for supergravity as well. We have also discussed the situation when there are more than one singlet chiral superfields. Simple conditions on the Kähler potential control whether or not the decoupling of the sgoldstino will make them decouple as well. The limit of an infinitely heavy sgoldstino nevertheless makes the Kähler potential separable thus strongly suppressing the sgoldstino-matter mixing.

However, there are supersymmetry breaking mechanisms that are not captured by the conventional non-linear sigma models disussed here. These cases include for example higher derivative supergravities. SUSY breaking in such theories, the identification of the (constraint) superfield goldstino belongs and the possibility of the sgoldstino decoupling are some problems that should be answered. In addition, further work is needed for understanding the effects of the sgoldstino decoupling on MSSM matter, in particular in the supergravity context.

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APPENDIX

We explicitly write below the supergravity theory that contains the goldstino. By integrating out the gravity multiplet auxiliary fields (M, b_n) and imposing the condition $A = \frac{\chi^2}{2F}$, the component form of a general chiral model coupled to supergravity with the generalized Kähler potential given by (3.46) turns out to be

$$
\begin{aligned}
e^{-1}\mathcal{L} = &-\frac{1}{2}R + \frac{1}{2}\varepsilon^{mnpq}\bar{\psi}_m\bar{\sigma}_n[\partial_p\psi_q + \psi_q\omega_p] - \frac{1}{2}\varepsilon^{mnpq}\psi_m\sigma_n[\partial_p\bar{\psi}_q + \bar{\psi}_q\omega_p] + \frac{i}{4}\varepsilon^{mnpq}\bar{\psi}_m\bar{\sigma}_n\psi_q(\chi\sigma_p\bar{\chi}) \\
&+ \frac{1}{4}\varepsilon^{mnpq}\bar{\psi}_m\bar{\sigma}_n\psi_q\left\{[\frac{\bar{\chi}^2}{2\bar{F}} + a]\partial_p\frac{\chi^2}{2F} - [\frac{\chi^2}{2F} + a]\partial_p\frac{\bar{\chi}^2}{2\bar{F}}\right\} - \partial\frac{\chi^2}{2F}\partial\frac{\bar{\chi}^2}{2\bar{F}} \\
&- \frac{i}{2}\chi\sigma^m(\partial_m - \omega_m)\bar{\chi} - \frac{i}{2}\bar{\chi}\bar{\sigma}^m(\partial_m - \omega_m)\chi - \frac{1}{8}\chi^2\bar{\chi}^2 - \frac{i}{4}\chi\sigma^m\bar{\chi}(a\partial_m\frac{\chi^2}{2F} - a\partial_m\frac{\bar{\chi}^2}{2\bar{F}}) \\
&- \frac{i}{2}\varepsilon^{klmn}(\chi\sigma_k\bar{\chi})(\psi_l\sigma_m\bar{\psi}_n) - \frac{1}{2}\psi_m\chi\,\bar{\psi}^m\bar{\chi} - \frac{1}{\sqrt{2}}(\bar{\psi}_m\bar{\sigma}^n\sigma^m\bar{\chi})\partial_n\frac{\chi^2}{2F} - \frac{1}{\sqrt{2}}(\psi_m\sigma^n\bar{\sigma}^m\chi)\partial_n\frac{\bar{\chi}^2}{2\bar{F}} \\
&- e^{\frac{b}{2}}\left\{1 + \frac{a\chi^2}{4F} + \frac{a\bar{\chi}^2}{4\bar{F}} + (1 + \frac{a^2}{2})\frac{\chi^2\bar{\chi}^2}{8F\bar{F}}\right\}[\psi_m\sigma^{mn}\psi_n + \bar{\psi}_m\bar{\sigma}^{mn}\bar{\psi}_n] \\
&+ \frac{i}{\sqrt{2}}e^{\frac{b}{2}}[a + (\frac{a^2}{2} + 1)\frac{\bar{\chi}^2}{2\bar{F}}](\bar{\psi}_m\bar{\sigma}^m\chi) + \frac{i}{\sqrt{2}}e^{\frac{b}{2}}[a + (\frac{a^2}{2} + 1)\frac{\chi^2}{2F}](\psi_m\sigma^m\bar{\chi}) \\
&+ 3e^b(1 + \frac{a\chi^2}{2F} + \frac{a\bar{\chi}^2}{2\bar{F}} + [1 + a^2]\frac{\chi^2\bar{\chi}^2}{4F\bar{F}}) - \frac{1}{2}e^{\frac{b}{2}}(a^2\chi^2 + a^2\bar{\chi}^2 + [2a + \frac{a^3}{2}]\frac{\chi^2\bar{\chi}^2}{2\bar{F}} + [2a + \frac{a^3}{2}]\frac{\chi^2\bar{\chi}^2}{2F}) \\
&+ Fe^{\frac{b}{2}}\left\{a + \frac{a^2\chi^2}{4F} + (1 + \frac{a^2}{2})\frac{\bar{\chi}^2}{2\bar{F}} + (a + \frac{a^3}{4})\frac{\chi^2\bar{\chi}^2}{4F\bar{F}}\right\} \\
&+ \bar{F}e^{\frac{b}{2}}\left\{a + \frac{a^2\bar{\chi}^2}{4\bar{F}} + (1 + \frac{a^2}{2})\frac{\chi^2}{2F} + (a + \frac{a^3}{4})\frac{\chi^2\bar{\chi}^2}{4F\bar{F}}\right\} + F\bar{F}. \qquad (4.1)
\end{aligned}
$$

The equations of motion for the auxiliary field F are easily solved iteratively and we have

$$
\bar{F} = -e^{\frac{b}{2}}a + 2\Omega B\bar{A} + \Omega\Gamma - e^{\frac{b}{2}}(1 + \frac{a^2}{2})\bar{A} + E\Omega\bar{\chi}^2 + 2\Omega H^p\partial_p\bar{A} + \frac{1}{2}\Omega\partial^2\bar{A} + \Omega\bar{Z}_{\dot{\rho}}\bar{\chi}^{\dot{\rho}} + \Omega\bar{O}^n_{\dot{\rho}}\partial_n\bar{\chi}^{\dot{\rho}} \qquad (4.2)
$$

where

$$
\begin{aligned}
A &= \frac{\chi^2}{2F} = -\frac{\chi^2}{2e^{\frac{b}{2}}a} - \frac{\bar{\Gamma}}{2e^{2b}a^4}\chi^2\bar{\chi}^2 + \frac{1}{8e^{\frac{5b}{2}}a^5}\chi^2\bar{\chi}^2\partial^2\chi^2 - \frac{\bar{\Gamma}}{2e^{2b}a^4}\chi^2\bar{\chi}^2 O^{n\rho}\partial_n\chi_\rho, \\
\Omega &= \frac{\chi^2}{F^2} = \frac{\chi^2}{2e^{\frac{b}{2}}a} + \frac{2\bar{\Gamma}}{e^{\frac{5b}{2}}a^5}\chi^2\bar{\chi}^2 - \frac{1}{2e^{3b}a^6}\chi^2\bar{\chi}^2\partial^2\chi^2 + \frac{2\bar{\Gamma}}{e^{\frac{5b}{2}}a^5}\chi^2\bar{\chi}^2 O^{n\rho}\partial_n\chi_\rho \qquad (4.3)
\end{aligned}
$$

and

$$
\begin{aligned}
B &= -\frac{1}{16}\varepsilon^{mnpq}\bar{\psi}_m\bar{\sigma}_n\psi_q\partial_p\mathrm{ln}e - \frac{1}{16}\varepsilon^{mnpq}[\bar{\psi}_m\bar{\sigma}_n\partial_p\psi_q + \partial_p\bar{\psi}_m\bar{\sigma}_n\psi_q], \\
&\quad -\frac{1}{8}e^{\frac{b}{2}}(1 + \frac{a^2}{2})[\psi_m\sigma^{mn}\psi_n + \bar{\psi}_m\bar{\sigma}^{mn}\bar{\psi}_n] + \frac{3}{4}e^b(1 + a^2) - \frac{1}{4}e^b(1 + \frac{a^2}{2})^2, \\
\Gamma &= -\frac{1}{4}e^{\frac{b}{2}}a[\psi_m\sigma^{mn}\psi_n + \bar{\psi}_m\bar{\sigma}^{mn}\bar{\psi}_n] + e^b a - \frac{1}{4}e^b a^3
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{1}{8}a\varepsilon^{mnpq}[\bar{\psi}_m\bar{\sigma}_n\partial_p\psi_q+\partial_p\bar{\psi}_m\bar{\sigma}_n\psi_q]-\frac{1}{8}a\varepsilon^{mnpq}\bar{\psi}_m\bar{\sigma}_n\psi_q\partial_p\mathrm{ln}e\,,\\
H^p &= -\frac{1}{8}\varepsilon^{mnpq}\bar{\psi}_m\bar{\sigma}_n\psi_q+\frac{1}{4}\partial^p\mathrm{ln}e\,,\\
E &= -\frac{1}{4}ae^{\frac{b}{2}}-\frac{1}{16}a^3e^{\frac{b}{2}},\\
\bar{Z}_{\dot{\rho}} &= \frac{1}{2\sqrt{2}}(\bar{\psi}_m\bar{\sigma}^n\sigma^m)_{\dot{\rho}}\partial_n\mathrm{ln}e+\frac{1}{2\sqrt{2}}(\partial_n\bar{\psi}_m\bar{\sigma}^n\sigma^m)_{\dot{\rho}}\\
& +\frac{i}{2\sqrt{2}}e^{\frac{b}{2}}(1+\frac{1}{2}a^2)(\psi_m\sigma^m)_{\dot{\rho}}+\frac{i}{8}a\partial_m\chi^\rho\sigma^m_{\rho\dot{\rho}},\\
\bar{O}^n_{\dot{\rho}} &= \frac{1}{2\sqrt{2}}(\bar{\psi}_m\bar{\sigma}^n\sigma^m)_{\dot{\rho}}.
\end{aligned}
\tag{4.4}
$$

By inserting (4.2) into (4.1), one finds the non-linear representation of local supersymmetry, which on top of the terms inside (2.37) also contains interactions with the gravitino now.

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